Importance Weighted Autoencoder (IWAE)

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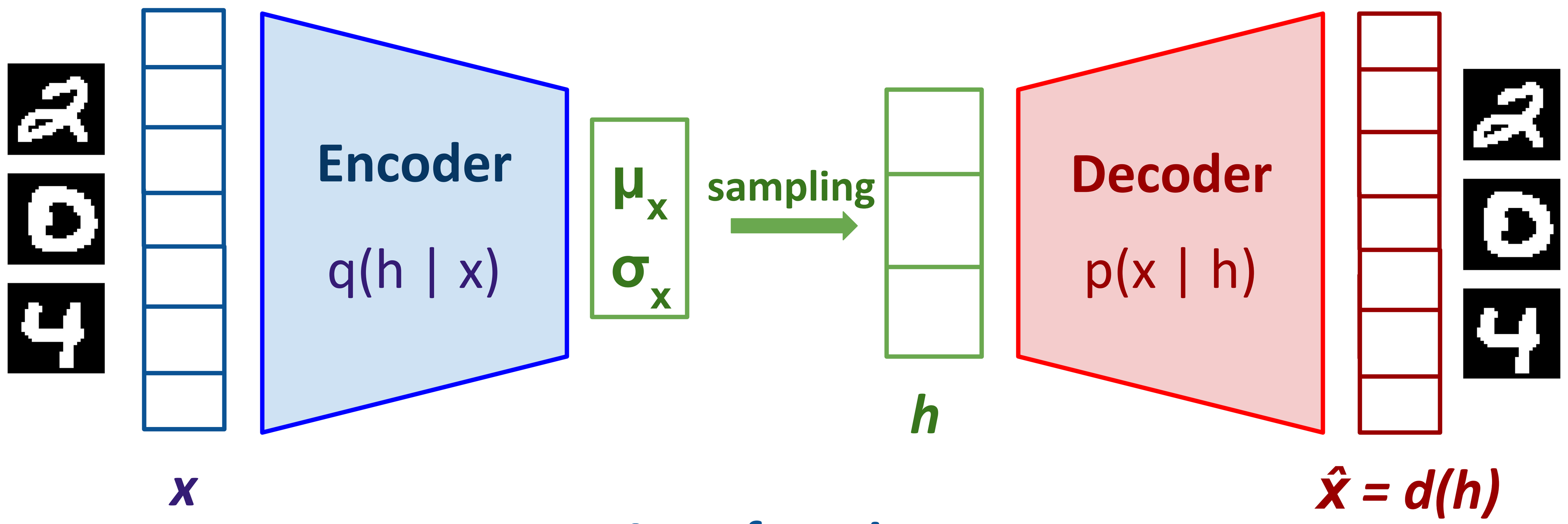
Architecture



Loss functions

Classical VAE

$$\mathcal{L}_{VAE}(x) = \mathbb{E}_{q(h|x)}\left[\log\left(\frac{p(x,h)}{q(h|x)}\right)\right] \cong \sum_{\substack{h_1,\dots,h_k \\ \sim q(h|x)}} \frac{1}{k}\log\left(\frac{p(x,h_i)}{q(h_i|x)}\right)$$

IWAE

$$\mathcal{L}^k_{IWAE}(x) = \mathbb{E}_{\substack{h_1,\dots,h_k \\ \sim q(h|x)}}\left[\log\left(\frac{1}{k}\sum_{i=1}^{k} \frac{p(x,h_i)}{q(h_i|x)}\right)\right] \cong \sum_{\substack{h_1,\dots,h_k \\ \sim q(h|x)}} \log\left(\frac{1}{k}\sum_{i=1}^{k}\frac{p(x,h_i)}{q(h_i|x)}\right)$$

Analysis

$$\log\left(p(x)\right) \geq \mathcal{L}^k_{IWAE}(x) \geq \mathcal{L}_{VAE}(x)$$
$$= \log\left(p(x)\right) - D_{KL}\left(q(h|x)||p(h|x)\right)$$

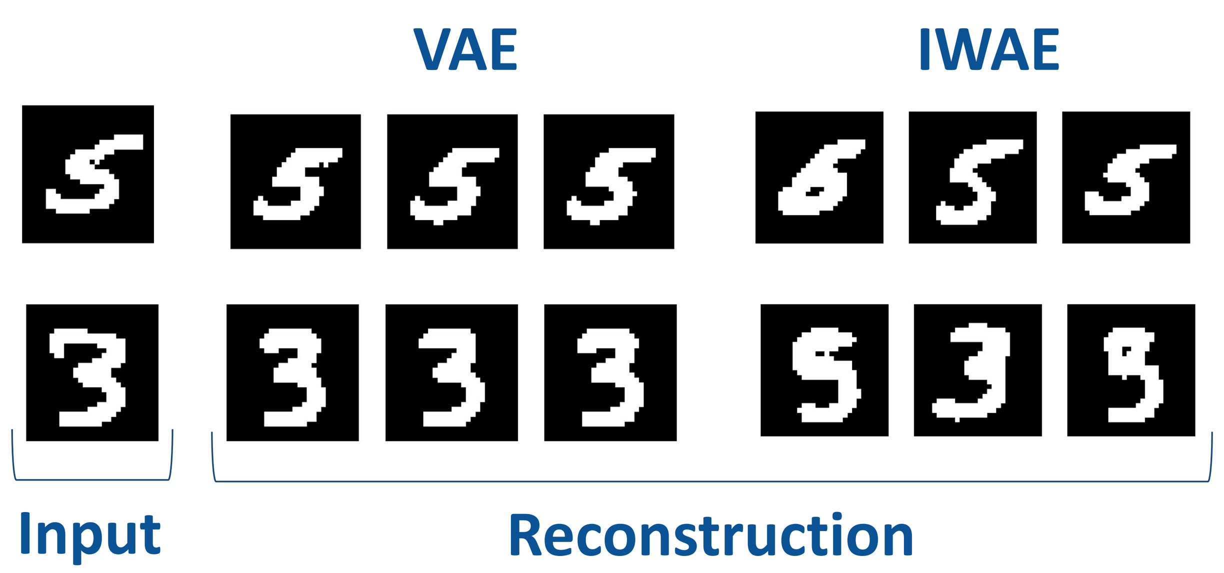
Weaker constraints on the posterior approximation

Pros

- ✔ Tighter lower-bound on the log-likelihood
- ✔ Richer latent representations

Cons

- ✘ Loss in encoding-decoding power

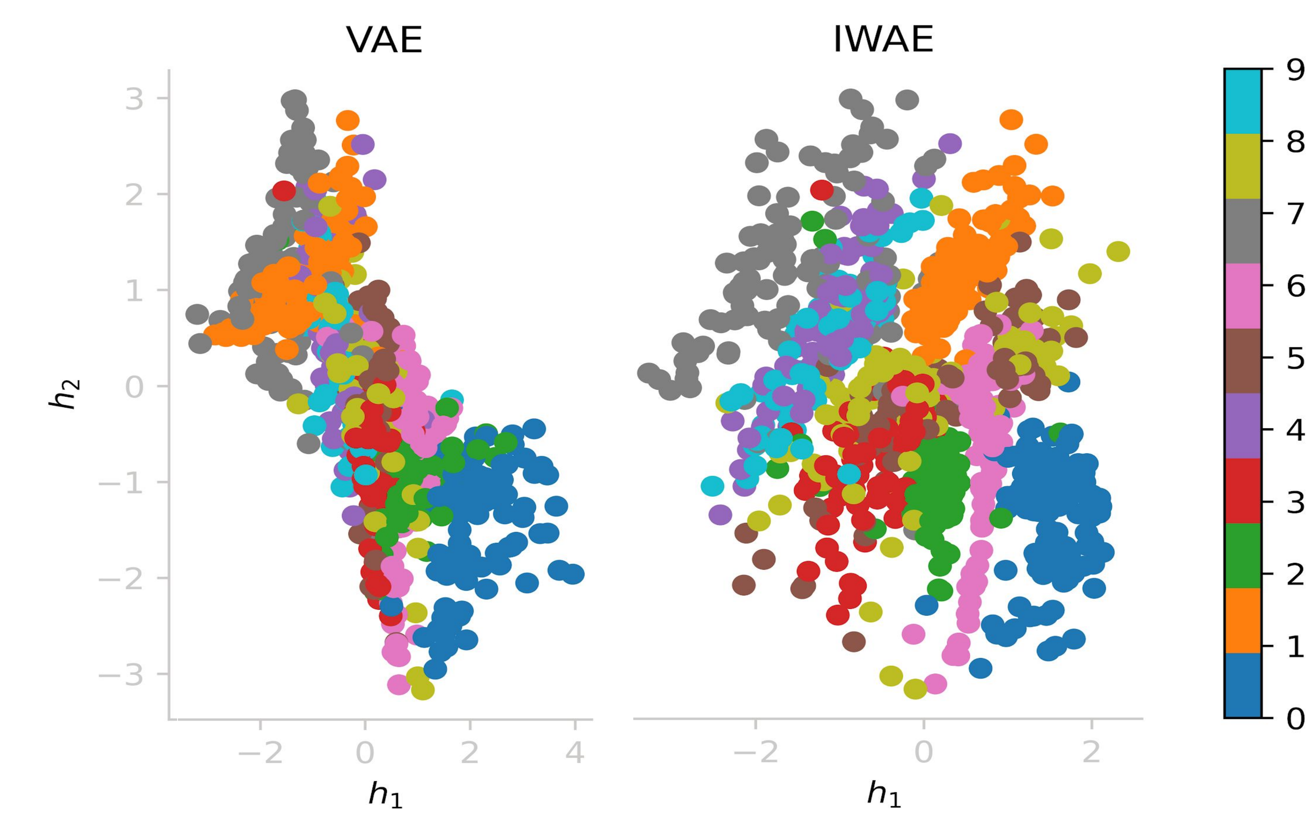


MNIST results

	VAE		IWAE	
k	Log-likelihood	Reconstruction Loss	Log-likelihood	Reconstruction Loss
1	-89.87	71.87	-89.87	71.87
5	-89.85	72.09	-87.61	71.63
50	-90.81	74.15	-87.13	78.36

Log-likelihood and reconstruction loss for models with one stochastic layer on the fixed binarization MNIST dataset [1]

IWAE learns a more spread-out latent representation



Also addressed in our report

- Generalisation to N stochastic layers
- Other datasets (OMNIGLOT)
- Investigation of the true latent space representation dimension
- Similar loss functions: $\mathcal{L}^k_p(x) = \mathbb{E}_{\substack{h_1,\dots,h_k \\ \sim q(h|x)}}\left[\log\left(\left[\frac{1}{k}\sum_{i}\left(\frac{p(x,h_i)}{q(h_i|x)}\right)^p\right]^{\frac{1}{p}}\right)\right]$

References

1. https://www.tensorflow.org/datasets/catalog/mnist
2. Kingma, D. P., and Welling M., "Auto-encoding variational bayes." arXiv preprint arXiv:1312.6114 (2013).
3. Yuri B., Roger G. and Ruslan S., "Importance Weighted Autoencoders" arXiv:1509.00519 (2015)

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